IDENTIFICATION OF PARAMETERS OF CONCRETE DAMAGE PLASTICITY CONSTITUTIVE MODEL

The paper presents a method and requirements of the material parameters identification for concrete damage plasticity constitutive model. The laboratory tests, which are necessary to identify constitutive parameters of this model have been presented. Two standard applications have been shown that test the constitutive model of the concrete. The first one is the analysis of the three-point bending single-edge notched concrete beam specimen. The second presents the four-point bending single-edge notched concrete beam specimen under static loadings.

In conclusion, the comparison of crack patterns in the numerical and laboratory [2,9] tests has been presented and discussed.

Keywords: Concrete Damage Plasticity (CDP), identification of parameters, failure and fracture of concrete

1. INTRODUCTION

The identification of constitutive parameters, that describe the material properties is fundamental. A numerical strategy for solving any boundary value problem (BVP) with location of fracture should consider a complex constitutive modelling. If structural material such as concrete is taken into account, it is necessary to identify a large number of parameters. The notion of concrete constitutes a wide range of materials, whose properties are quantitatively and qualitatively different for typical tests (compression and tension). Recently, modelling of failure and fracture has become one of the fundamental issues in structural mechanics.
particularly in concrete structures. In this paper, a scalar variable is used to model
the failure (in both compression and tension). The main task in failure description
is the recognition of crack patterns. Concrete Damage Plasticity (CDP) is one of
the possible constitutive model. In this paper, the typical laboratory tests of concrete,
that are necessary to identify the process, have been proposed.

Introduced by Kachanov [5] and further developed by Rabotnov [8] and
others [1,5,7,8], the constitutive equation of material with scalar isotropic dam-
age takes the following form:

$$\sigma = (1 - d) D^e_0 : (\varepsilon - \varepsilon^{pl}) = D^e_0 : (\varepsilon - \varepsilon^{pl}),$$

(1.1)

where $\sigma$ is Cauchy stress tensor, by $d$ is the scalar stiffness degradation vari-
able, respectively, $\varepsilon$ is the strain tensor, $D^e_0$ the initial (undamaged) elastic
stiffness of the material, while $D^e = (1 - d) D^e_0$ is the degraded elastic stiffness
tensor. The effective stress tensor is defined as:

$$\bar{\sigma} = D^e_0 : (\varepsilon - \varepsilon^{pl}),$$

(1.3)

where $\varepsilon^{pl}$ is the plastic strain. In the formulation, it is necessary to propose the
evolution of the scalar degradation variable:

$$d = d(\sigma, \varepsilon^{pl})$$

(1.2)
governed by a set of the effective stress tensor $\bar{\sigma}$ and hardening (softening) vari-
ables $\tilde{\varepsilon}^{pl}$. In CDP model, the stiffness degradation is initially isotropic and de-
defined by degradation variable $d_c$ in a compression zone and variable $d_t$ in a
tension zone.

Thus, finally, the Cauchy stress tensor is related to the effective stress ten-
sor $\bar{\sigma}$ through the scalar degradation parameter $(1-d)$:

$$\sigma = (1 - d) \bar{\sigma}.$$  

(1.4)

Damage states in tension and compression are characterized independ-
ently by two hardening variables, $\tilde{\varepsilon}^{pl}_{\varepsilon}$ and $\tilde{\varepsilon}^{pl}_{\varepsilon}$, which are referred to equivalent
plastic strains in tension and compression, respectively. The evolution of the
hardening variables is given by the following expression:

$$\tilde{\varepsilon}^{pl} = \begin{bmatrix} \tilde{\varepsilon}^{pl}_{\varepsilon} \\ \tilde{\varepsilon}^{pl}_{\varepsilon} \end{bmatrix}$$

and

$$\varepsilon^{pl} = h(\sigma, \varepsilon^{pl}) \cdot \tilde{\varepsilon}^{pl}.$$  

(1.5)
Cracking (tension) and crushing (compression) in concrete are represented by increasing values of the hardening (softening) variables. These variables control the evolution of the yield surface and the degradation of the elastic stiffness.

The yield function represents a surface in effective stress space which determines the states of failure or damage. For the inviscid plastic-damage model the yield function arrives at:

$$ F\left(\bar{\sigma}, \bar{\varepsilon}^p\right) \leq 0. $$

Plastic flow is governed by a flow potential function $G(\bar{\sigma})$ according to nonassociative flow rule:

$$ \dot{\varepsilon}^p = \dot{\lambda} G(\bar{\sigma}). $$

The plastic potential function $G$ is also defined in the effective stress space.

Constitutive model of concrete (CDP) is one of the possible concepts. The behaviour of concrete depends on material parameters, which are identified in the paper.

2. ESSENTIAL CONSTITUTIVE PARAMETERS FOR IDENTIFICATION AND THE NECESSARY LABORATORY TESTS

The laboratory tests, which are necessary for the identification process of constitutive parameters $\beta$, $m$, $f$ and $\gamma$ have been presented in this section. The parameters $\beta$ and $m$ are used to describe the shape of the flow potential function, while $f$ and $\gamma$ are responsible for the shape of the yield function.

The identification procedure is realizeded for concrete, class B50. The sources of the experimental results are the works [2,4,9] and experiments (uniaxial compression and tension tests) elaborated at home Institute. A curves for both uniaxial compression and tension tests and for multiaxial tests are presented in Figs 1-3.

The Kupfer’s curve (i.e. failure curve in plane stress [6]) for concrete class B50 is necessary. The stress-strain curves in triaxial state of stress are presented in Fig. 3. The last test is a superposition of two states: 1) hydrostatic state of stress (effective compressive stresses $p$ equal 0.0, 6.9 and 13.8 MPa) and 2) uniaxial compression in direction of $\sigma_{33}$. 
Fig. 1. Uniaxial compression test of concrete, class B50 – experimental curve (Home Institute)

Fig. 2. Uniaxial tension test of concrete, class B50 – experimental curve (Home Institute)

Fig. 3. Triaxial compression of concrete, class B50 – experimental curve [4]
3. THE PROCEDURE OF CONSTITUTIVE PARAMETERS IDENTIFICATION FOR CDP MODEL

The fundamental group of the constitutive parameters consists of four values, which identify the shape of the flow potential surface and the yield surface. In this model for the flow potential $G$, the Drucker-Prager hyperbolic function is accepted in the form:

$$G = \sqrt{(f_c - m \cdot f_t \cdot \tan \beta)^2 + \bar{q}^2} - \bar{p} \cdot \tan \beta \cdot \sigma,$$  \hspace{1cm} (3.1)

where $f_t$ and $f_c$ are the uniaxial tensile and compressive strengths of concrete, respectively. $\beta$ is the dilation angle measured in the $p-q$ plane at high confining pressure, while $m$ is an eccentricity of the plastic potential surface. The flow potential surface is defined in the $p-q$ plane, where $\bar{p} = -\frac{1}{3} \bar{\sigma} \cdot I$ is the effective hydrostatic stress and $\bar{q} = \sqrt{\frac{1}{2} \bar{S} \cdot \bar{S}}$ is the Mises equivalent effective stress, while $\bar{S}$ is the deviatoric part of the effective stress tensor $\bar{\sigma}$.

The nonassociative flow rule, which is used here requires a loading surface definition. The plastic-damage concrete model uses a yield condition based on the loading function (3.2) proposed by Lubliner in [7] in the form:

$$F = \frac{1}{1-\alpha} \left[ \bar{q} - 3 \cdot \alpha \cdot \bar{p} + \theta(\bar{v}_{pl}) \left( \bar{\sigma}_{\text{max}} \right) - \gamma (\bar{\sigma}_{\text{max}}) \right] - \bar{\sigma}_{\text{c}} (\bar{v}_{pl}).$$ \hspace{1cm} (3.2)

The shape of loading surface in the deviatoric plane is determined by parameter $\gamma$, while the parameter $\alpha$ is calculated based on Kupfer’s curve. $\bar{\sigma}_{\text{max}}$ is the algebraically maximum eigenvalue of $\bar{\sigma}$. The Macauley bracket $\langle \cdot \rangle$ is defined by $\langle x \rangle = \frac{1}{2} (|x| + x)$. The function $\theta(\bar{v}_{pl})$ is given as

$$\theta(\bar{v}_{pl}) = \frac{\bar{\sigma}_c (\bar{v}_{pl})}{\bar{\sigma}_t (\bar{v}_{pl})} (1-\alpha) - (1+\alpha),$$ \hspace{1cm} (3.3)

where $\bar{\sigma}_t$ and $\bar{\sigma}_c$ are the effective tensile and compressive cohesion stresses, respectively.

It is necessary to define the parameter:

$$\alpha = \frac{(f_{b0}/f_c) - 1}{2(f_{b0}/f_c) - 1}.$$ \hspace{1cm} (3.4)

The compressive strength under biaxial loading of concrete is denoted by $f_{b0}$. 

The parameter $\alpha$ depends on the ratio of the biaxial compressive strength and uniaxial compressive strength. Thus, the biaxial laboratory test [6] is necessary to define the value of $\alpha$. The procedure of identification of $m$ and $\beta$ parameters will be presented in this section.

It is clearly seen, that the behaviour of concrete depends on four constitutive parameters. Other parameters such as tensile uniaxial strength and uniaxial or biaxial compressive strength of concrete should be taken from experimental curves. Moreover, the parameter $\gamma$ should be defined based on the full triaxial tests of concrete. Identification of that parameter is possible only if the full triaxial compression tests of concrete are done. In accordance to [7] the parameter $\gamma$ is prescribed in the form:

$$\gamma = \frac{3(1 - \rho)}{2\rho + 3},$$

where the coefficient:

$$\rho = \left( \frac{J_2^{TM}}{J_2^{CM}} \right)$$

is defined at a given state $\bar{p}$.

In Eqn. (3.6), designate $TM$ and $CM$ mean, respectively, the „tensile meridian” ($\sigma_1 > \sigma_2 = \sigma_3$) and the „compressive meridian” ($\sigma_1 = \sigma_2 > \sigma_3$) in the yield surface. Typical values range for $\rho$ are between 0.64 to 0.8 [7].

![Fig. 4. Dependence $\sigma - \varepsilon$ in compression for CDP model](image)

Based on experimental $\sigma - \varepsilon$ curves for both uniaxial tension and compression, it is possible to definite the dependence between stress – cracking
Identification of parameters of concrete damage plasticity constitutive model

strain ($\varepsilon^c_t$) in uniaxial tension and stress – crushing strain ($\varepsilon^c_c$) in uniaxial compression. Figs 4, 5 show, which values in CDP model are interpreted as the cracking strain ($\varepsilon^c_t$) and the crushing strain ($\varepsilon^c_c$).

Fig. 5. Dependence $\sigma - \varepsilon$ in tension for CDP model

Fig. 6. The Kupfer’s curve for concrete class B50 [6]

It is additionally supposed, that a range of concrete elasticity is 0.3 $f_c$ in compression and 0.7 $f_t$ in tension. In the elastic zone the Young’s modulus equals 19.7 GPa and the Poisson’s ratio equals 0.19. The damage variables $d_t$ and $d_c$ are defined based on plots presented in Figs 1, 2, 4, 5. The dependencies $d_t$ on
cracking strain (\(\varepsilon_{\text{ck}}\)) and \(d_c\) on crushing strain (\(\varepsilon_{\text{cin}}\)) should be determined. To identify the shape of the flow potential and the loading surfaces the strength of concrete for uniaxial tests, both compressive and tensile were used. Both surfaces are defined by the four parameters: \(\beta\), \(m\), \(f\) and \(\psi\). The value 0.666 is accepted for parameter \(\psi\). The shape of the loading surface in deviatoric plane is determined by this parameter. This paper does not discuss the identification procedure for \(\psi\) parameter, because the authors had no the results of full triaxial tests of concrete. The Kupfer’s curve is used to define the parameter \(\psi = f_{10}/f_c\), as is shown in Fig. 6. The parameter \(\alpha\) is computed based on \(f_c\), according to Eqn. 3.4.

Tab. 1. Coordinates of the principal stress \(\sigma_1, \sigma_2, \sigma_3\)

<table>
<thead>
<tr>
<th>No</th>
<th>(p_i) [MPa]</th>
<th>(q_i) [MPa]</th>
<th>(\sigma_1) [MPa]</th>
<th>(\sigma_2) [MPa]</th>
<th>(\sigma_3) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.84</td>
<td>0</td>
<td>-2.84</td>
<td>-2.84</td>
<td>-2.84</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>30</td>
<td>0,000</td>
<td>0,000</td>
<td>20.2</td>
</tr>
<tr>
<td>3</td>
<td>19.9</td>
<td>39</td>
<td>6.9</td>
<td>6.90</td>
<td>35.9</td>
</tr>
<tr>
<td>4</td>
<td>30.8</td>
<td>51</td>
<td>13.8</td>
<td>13.80</td>
<td>54.8</td>
</tr>
</tbody>
</table>

![Fig. 7 The meridian plane of the flow potential surface](image-url)
The parameters $m$ and $\beta$, which determine the shape of the flow potential surface are accepted to fulfil the best fitting of the curve (in the meridian plane) to the experimental results; (see Fig. 7).

Thus, the minimization of the error of identification for four points is necessary to determine the two parameters $m$ and $\beta$. It is the error in the meaning of the least square method. The points in $p-q$ space are presented in Tab. 1 and they correspond with the ends of experimental curves in Fig. 3. The principal components of stress state are also indicated in Tab. 1 [4]. An identification task requires the error minimization of the following in the form of:

$$f = \sum_{i=1}^{4} \left( q_i - q(p_i, m, \beta, f_1) \right)^2 \rightarrow \text{min}. \quad (3.7)$$

In Tab. 1, the point coordinates $(-2.84,0.0)$ of the point don’t follow directly Fig. 3. This point represents the triaxial tension of concrete and in principal, its coordinate influences the value of parameter $m$.

In Fig. 8, the sensitivity (to the change of parameters $m$ and $\beta$) of functional (3.7) is presented. Subsequently, that functional is subjected to minimization with limitation. It is supposed, that $m$ ranges $\langle 1.11 \rangle$ and $\beta$ ranges $\langle 0.6, 0.75 \rangle$. Minimise of error brings to the values of parameters $m$ and $\beta$, in sequence equal 1.0 and 0.68 radian (38 degree). It is clearly seen, that the error of identification is sensitive to the change of parameter $\beta$. The conjugate gradient method in environment of MathCad program is used for the optimization process.

Fig. 8. The shape of functional $f$ in $m, \beta$ space
The hardening and softening rule and the evolution of the scalar damage variable for compression and tension are presented in Tab. 2. Both depend on the crushing or cracking strains.

Tab. 2. The material parameters of CDP model for concrete class B50

<table>
<thead>
<tr>
<th>Material’s parameters</th>
<th>B50</th>
<th>The parameters of CDP model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete elasticity</td>
<td></td>
<td>( \beta )</td>
</tr>
<tr>
<td>( E [\text{GPa}] )</td>
<td>19.7</td>
<td>( f = f_{00} / f_c )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.19</td>
<td>( \gamma )</td>
</tr>
</tbody>
</table>

Concrete compression hardening

<table>
<thead>
<tr>
<th>Stress [MPa]</th>
<th>Crushing strain [-]</th>
<th>DamageC [-]</th>
<th>Crushing strain [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20.197804</td>
<td>0.0000747307</td>
<td>0.0</td>
<td>0.0000747307</td>
</tr>
<tr>
<td>30.000609</td>
<td>0.0000988479</td>
<td>0.0</td>
<td>0.0000988479</td>
</tr>
<tr>
<td>40.303781</td>
<td>0.000154123</td>
<td>0.0</td>
<td>0.000154123</td>
</tr>
<tr>
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<td>0.0</td>
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<tr>
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<tr>
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</tr>
<tr>
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<td>0.011733119</td>
<td>0.894865</td>
<td>0.011733119</td>
</tr>
</tbody>
</table>

Concrete tension stiffening

<table>
<thead>
<tr>
<th>Stress [MPa]</th>
<th>Cracking strain [-]</th>
<th>DamageT [-]</th>
<th>Cracking strain [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.99893</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.842</td>
<td>0.00003333</td>
<td>0.0</td>
<td>0.00003333</td>
</tr>
<tr>
<td>1.86981</td>
<td>0.000160427</td>
<td>0.406411</td>
<td>0.000160427</td>
</tr>
<tr>
<td>0.862723</td>
<td>0.000279763</td>
<td>0.69638</td>
<td>0.000279763</td>
</tr>
<tr>
<td>0.226254</td>
<td>0.000684593</td>
<td>0.920389</td>
<td>0.000684593</td>
</tr>
<tr>
<td>0.056576</td>
<td>0.00108673</td>
<td>0.980093</td>
<td>0.00108673</td>
</tr>
</tbody>
</table>

Eventually, for the identification of the constitutive parameters of CDP model the following laboratory tests are necessary:

- the uniaxial compression,
- the uniaxial tension,
- the biaxial failure in plane state of stress (the Kupfer’s curve for concrete class B50),
- the triaxial test of concrete (superposition of the hydrostatic state of stress and the uniaxial compression stress).

These tests are necessary to identify the parameters, which determine the shape of the flow potential surface in the deviatoric and meridian plane and the
evolution rule of the material parameters (the hardening and the softening rule in tension and compression).

4. NUMERICAL COMPUTATIONS

In the computations of the standard applications we used the finite element code, implemented in the environment of ABAQUS/Explicit. The models and the computations lead to the estimation of the nucleation and the evolution of fracture in bending beams with notches (three-point and four-point bending). The numerical results are in agreement with the experiments [2,9]. The scalar damage variable in tension is used to compare crack patterns for the numerical and experimental models.

4.1. The three-point bending beam

The applications of CDM model to the selected BVPs and the comparisons of the results (computations with experiments [2,9]) prove the usefulness of the proposed constitutive model. The three-point bending single-edge notched concrete beam is used to compare the numerical results with the laboratory tests [2]. The geometry of the three-point bending beam specimen is presented in Fig. 9. All dimensions are given in millimeters. The thickness of the beam equals 100 mm.

![Fig. 9. The geometry of three-point bending single-edge notched concrete beam [2]](image)

The finite element mesh used in computations is shown in Fig. 10. The numerical model consists of 465 three-node linear plane stress elements. In the investigated case, the mesh in the vicinity of the notch-tip is finer and its density is doubled in comparison with the rest of the model. The other (also finer) meshes were discussed and the obtained crack patterns were similar, which proves the appropriateness of its acceptance.
Fig. 10. The finite elements mesh

Fig. 11. The comparison of crack patterns for three-point bending single-edge notched beam, a) beam with CDP numerical model, b) the fracture path, which is observed in experiment [2], c) the qualitative comparison of the plots: force-displacement for discussed cases
The calculated crack pattern is similar to that observed in the experimental model [2]. The qualitative comparison of the results is presented in Fig. 11c.

The single dominant crack appeared in the concrete three-point bending beam specimen. The shape of the fracture zone location is shown in Fig. 11a. In the experiment presented by Davies in [2], the crack pattern is similar to the obtain numerical results, Fig. 11b. The results, which are presented in Fig. 11a) and b), correspond to the respective level of load and are shown in Fig. 11c.

4.2. The four-point bending beam

The following numerical experiment, which can verify the constitutive equation (CDP) is the four-point bending single-edge notched beam, as shown in Fig. 12. This test verifies the concrete CDP model for the case of dominant shearing. The one parameter loading is distributed in a specific way- as shown in Fig. 12. In the numerical simulation the force is applied by the rigid surface to the concrete specimen. The interval between the force and the central support is ten times shorter than between the force and the left support. The constraint has a kinematic character. Between the concrete beam specimen and the rigid surface, where the load is applied, contact conditions are realized. Thus, the pressure forces are distributed in a specific way as in the experiment.

The computations were performed for the different meshes. The first analysis with dominant three-node elements and the second one with dominant four-node elements have been computed. For both cases, the mesh near the notch region is finer and the dimensions of elements are three times smaller. In Fig. 13 the mesh with the dominant three-node, plane stress elements is presented. The thickness of the beam equals 100 mm.

![Fig. 12. The geometry of four-point bending single-edge notched concrete beam [7]](image)

The crack pattern, which was observed in the experiment, is presented in Fig. 14. The failure in beam specimens, analysed by Schlangen [9], propagates from the notch to the place of force application. The second mesh with the three-node and reduced integration elements was also used in the computations.
The comparison of the numerical results obtained for both meshes is presented in Fig. 16. The crack patterns for both numerical models are similar to that of the experimental beam, which was examined by Schlangen in [9]. Important is that the solution is unique and it does not depend on mesh size and type in a “pathologic” way, as is shown in Fig. 15. In the Figure the comparison of force – CMSD (the crack mouth sliding displacement) plots for two different meshes are shown. It can be recognized, that accepted space digitization does not influence the qualitative solution.

5. CONCLUSIONS

The examples have shown, that using CDP model enables a proper definition of the failure mechanisms in concrete elements. The CDP can be used to model the behaviour of concrete and the reinforced concrete structures and the other pre-stressed concrete structures in advanced states of loadings. Based on the criteria defined in this paper and the application of the laboratory tests, it is possible to identify the constitutive parameters of CDP model of concrete. The study also serves as a link between the real behaviour of concrete and its numerical modelling.
It is proved by examples, that the point of initiation and evolution of fracture is correctly estimated. The definition of anisotropy of concrete in compression and tension eventually leads to correct crack patterns. The CDP in comparison with the gradient models gives similar results [3].
The proposed model with the estimated constitutive parameters can successfully serve for analyses of the R-C structures in advanced states of stresses far beyond the limits defined in engineering codes.

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REFERENCES


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IDENTYFIKACJA PARAMETRÓW KONSTYTUTYWNYCH MODELU BETONU PLASTYCZNEGO ZE ZNISZCZENIEM

Streszczenie

W pracy przedstawiono metodę identyfikacji parametrów materiałowych betonu klasy B50. Przyjęto model matematyczny betonu plastycznego ze zniszczeniem. Określono, jakie testy laboratoryjne są niezbędne do identyfikacji parametrów
konstytutywnych tego modelu. Przetestowano użyteczność modelu konstytutywnego w wybranych dwóch zadaniach brzegowych (zginanie trójpunktowe oraz czteropunktowe belki betonowej z nacięciem). Porównano otrzymane mechanizmy zniszczenia ze znanymi wynikami eksperymentów laboratoryjnych [2, 9].

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